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Effective locomotion at multiple stride frequencies using proprioceptive feedback on a legged microrobot

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Abstract

Limitations in actuation, sensing, and computation have forced small legged robots to rely on carefully tuned, mechanically mediated leg trajectories for effective locomotion. Recent advances in manufacturing, however, have enabled in such robots the ability for operation at multiple stride frequencies using multi-degree-of-freedom leg trajectories. Proprioceptive sensing and control is key to extending the capabilities of these robots to a broad range of operating conditions. In this work, we use concomitant sensing for piezoelectric actuation with a computationally efficient framework for estimation and control of leg trajectories on a quadrupedal microrobot. We demonstrate accurate position estimation (<16% root-mean-square error) and control (<16% root-mean-square tracking error) during locomotion across a wide range of stride frequencies (10 Hz–50 Hz). This capability enables the exploration of two bioinspired parametric leg trajectories designed to reduce leg slip and increase locomotion performance (e.g. speed, cost-of-transport (COT), etc). Using this approach, we demonstrate high performance locomotion at stride frequencies (10 Hz–30 Hz) where the robot’s natural dynamics result in poor open-loop locomotion. Furthermore, we validate the biological hypotheses that inspired the trajectories and identify regions of highly dynamic locomotion, low COT (3.33), and minimal leg slippage (<10%).

1. Introduction

Terrestrial animals use a variety of complex leg trajectories to navigate natural terrains [1]. The choice of leg trajectory is often determined by a combination of morphological factors including posture [2], hip and leg kinematics [3], ankle and foot designs [4], and actuation capabilities (e.g. muscle mechanics [5, 6]). In addition, animals also modify their leg trajectories to meet performance requirements such as speed [7], stability [8, 9], and economy [10], as well as to adapt to external factors such as terrain type [11, 12] and surface properties [13, 14].

Inspired by their biological counterparts, large (body length (BL) ~ 100 cm) bipedal [15, 16] and quadrupedal [17–20] robots typically have two or more actuated degrees-of-freedom (DOF) per leg to enable complex leg trajectories. This dexterity is leveraged in a variety of control schemes to adapt to different environments and performance requirements. For example, optimization algorithms have been used to command leg trajectories to enable stable, dynamic locomotion on the Atlas bipedal [21] and HyQ quadrupedal [19] robots. Furthermore, the MIT Cheetah [18] relies on a hierarchical control scheme where the low-level controllers alter leg trajectories to directly modulate ground reaction forces.

However, as the robot’s size decreases, manufacturing and material limitations constrain the number of actuators and sensors. Consequently, a majority of medium (BL ~ 10 cm) [22] and small (BL ~ 1 cm) [23–25] legged robots have at most single DOF legs driven by a hip actuator. In such systems, leg trajectory is dictated by the transmission design, and these robots often rely on tuned passive dynamics to achieve efficient locomotion [26, 27]. Nevertheless, careful mechanical design allows these robots to demonstrate impressive capabilities, including high-speed running...
[28], jumping [29], climbing [30, 31], horizontal to vertical transitions [12], and confined space locomotion [14]. Recent work has also focused on developing whole-body locomotion control schemes for the autonomous operation of these small legged robots. These include controllers designed using stochastic kinematic models on the octopodal OctoRoACH [32] and using deep reinforcement learning on the hexapodal VelociRoACH [33] robots. However, these robots do not have the mechanical dexterity to actively vary the shape of the their leg trajectory and instead rely on mechanical tuning and inter-leg timing (i.e. gait) to achieve effective locomotion at a specific operating frequency. In contrast, the Harvard Ambulatory MicroRobot (HAMR, figure 1(a)) is able to independently control the fore-aft and vertical position of each leg using high-bandwidth piezoelectric bending actuators. This dexterity enables control over both the shape of individual leg trajectories and gait. Furthermore, HAMR is unique among legged robots in its ability to operate at a wide range of stride frequencies. Despite this dexterity, however, a lack of sensing and control has limited its operation to using feedforward sinusoidal voltage inputs resulting in elliptical leg trajectories [34, 35]. Though this approach has previously enabled rapid locomotion [36], high-performance operation (e.g. high speed, low COT, etc) has been limited to a narrow range of stride frequencies [37].

In this work, we leverage concomitant sensing for piezoelectric actuation (figure 1(b), [38]) and a computationally inexpensive estimator and controller (figure 1(c)) for tracking leg trajectories on a microrobot. The robot and concomitant sensors and discussed in section 2. We then describe the estimator (section 3) and controller (section 4) and include an important simplification, treating of ground contact as a perturbation. We leverage this capability to track two bioinspired parametric leg trajectories that modulate intra-leg timing, energy, and stiffness (section 5). We experimentally evaluate these trajectories (section 6), and demonstrate that our framework enables accurate estimation (section 7.1) and tracking (section 7.2) for our operating conditions (10 Hz–50 Hz). Furthermore, we find that these trajectories allow the robot to maintain locomotion performance in the body dynamics frequency regime by reducing leg slip, improving COT, and favorably utilizing body dynamics (section 8). We generalize these results across the range of operating stride frequencies in section 9, and the discuss implications of this work and potential future extensions in section 10.

2. Platform overview

This section describes the relevant properties of the microrobot (section 2.1) and the concomitant sensors (section 2.2).

2.1. Robot description

HAMR (figure 1(a)) is a 4.5 cm long, 1.43 g quadrupedal microrobot with eight independently actuated DOFs [39]. Each leg has two DOFs that are driven by optimal energy density piezoelectric bending actuators [40]. These actuators are controlled with AC voltage signals using a simultaneous drive configuration described by Karpelson et al [41]. A spherical-five-bar (SFB) transmission connects the two actuators to a single leg in a nominally decoupled manner: the swing actuator controls the leg’s fore-aft position, and the lift actuator controls the leg’s vertical position. A minimal-coordinate representation of the pseudo-rigid body dynamics of this robot has a configuration vector \( \mathbf{q} = [q^b, q^a]^T \in \mathbb{R}^{14} \) and takes the AC voltages signals \( u^a \in \mathbb{R}^8 \) as inputs. The configuration vector consists of the floating base position and orientation \((q^b \in \mathbb{R}^6)\), and the tip deflections of the eight actuators \((q^a \in \mathbb{R}^8)\). An alternative minimal-coordinate representation occasionally used in this work is \( q^{th} = [q^b, q^a]^T \in \mathbb{R}^{14} \). Here \( \mathbf{q}^a \in \mathbb{R}^8 \) is the vector of the four legs’ fore-aft \( (F) \) and vertical \( (V) \) positions, and it is related to \( \mathbf{q}^a \) by a one-to-one kinematic transformation.

2.2. Sensor design and dynamics

Eight off-board piezoelectric encoders provide measurements of actuator tip-velocities \((\dot{q}^a \in \mathbb{R}^8)[38]\). Though these sensors are currently off-board, an on-board implementation is straightforward as the components are both light (<10 mg) and small (<5 mm³). Previous work has shown that the tip-velocity of the \( i \)th actuator \((\dot{q}^a_i)\) is \( \alpha \) times the mechanical current \((i^m)\) produced by that actuator’s motion; that is,

\[
\dot{q}^a_i = \alpha i^m. \tag{1}
\]

Each encoder (figure 1(b)) measures the mechanical current by applying Kirchhoff’s law to the measurement circuit in series with a lumped-parameter electrical model of an actuator:

\[
i^m = \frac{V^m - V}{R_s} - \beta CV - \frac{V}{R}. \tag{2}
\]

The first term on the RHS of equation (2) is the total current drawn by an actuator computed from measurements of the voltage before \((V^m)\) and after \((V)\) a shunt resistor \((R_s = 75 \, \Omega)\). The actuator is modeled as a capacitor \((C)\), resistor \((R)\), and current source \((i^m)\) in parallel. The voltage and frequency dependent values of \( R \) and \( C \) have been computed for our operating conditions by Jayaram et al [38]. Finally, \( \beta \) is a blocking factor which accounts for imperfect measurements of \( C \), and is set to 1.57 as described by Jayaram et al [38].

3. Estimator design

We use the sensors described above in a proprioceptive estimator for leg position and velocity.
the continuous nonlinear difference equation:

\[ x_{t+1}^p = f(x_t^p, u_t^p), \]

where \( k \) is the time-step, \( x_t^p \) and \( u_t^p \) are the position and velocity of the swing and lift actuators, and \( u_t^p \) are the actuator drive voltages. A detailed derivation of \( f(x_t^p, u_t^p) \) is presented in Note S1. Instead of calculating the linear approximation of \( f(x_t^p, u_t^p) \) about a fixed point \((x_0^p, u_0^p)\), we use MATLAB’s subspace identification algorithm \texttt{n4sid} [48, 49] to determine a discrete-time second-order (four-state) linear system that minimizes the prediction error for the range of expected actuator deflections (±0.15 mm) and stride frequencies (10 Hz–50 Hz). While the accuracy of a local linear approximation decreases away from the fixed point, the identified model is accurate in an average sense across the range of expected operating conditions. The resulting discrete-time linear system has the form:

\[ x_{t+1} = A^p x_t + B^p (u_t^p - u_0^p) + w_t^p, \]

with \( A^p \in \mathbb{R}^{4 \times 4} \) and \( B^p \in \mathbb{R}^{4 \times 2} \). Moreover, the signal \( w_t^p \in \mathbb{R}^4 \) is zero-mean process noise with covariance \( W^p \). The \texttt{n4sid} algorithm determines the system matrices \((A^p, B^p)\) and noise covariance \((W^p)\) of the zero-mean process noise that minimize the squared prediction error in \( x_t^p - x_0^p \) when driven with voltages \( u_t^p - u_0^p \). We describe the identification process in evaluating the accuracy of the resulting model in Note S2. Finally, we note that \( x^p \) and \( u^p \) are subsets of \( x^a \) and \( u^a \) corresponding to the appropriate transmission, and the identical procedure is carried out to identify a process model for each transmission.

### 3.2. Measurement model

Since each piezoelectric encoder measurement is independent, the sensor dynamics (section 2.2) is inverted to form the measurement model for a single actuator. We start by combining equations (1) and (2) with a finite difference approximation of \( V \) to write a difference equation for \( \dot{y} \):

\[ \dot{q}_k = c_1 (V_{k-1}^m - V_k) - c_2 V_k - c_3 (V_k - V_{k-1}), \]

where \( c_1 = \alpha R^{-1}, \ c_2 = \alpha R^{-1}, \) and \( c_3 = \beta C^{-1}. \) Since equation (5) depends on the previous time-step, we also write a difference equation for \( \dot{q}_{k-1} \) using the same finite difference approximation for \( V_{k-1} \):

\[ \dot{q}_{k-1} = c_1 (V_{k-1}^m - V_{k-1}) - c_2 V_{k-1} - c_3 (V_{k-1} - V_{k-1}). \]

Combining equations (5) and (6) and solving for \( y_k^m = [V_k^m, V_{k-1}^m] \in \mathbb{R}^2 \) gives the measurement model:

\[ y_k^m = H^m x_k^m + D^m u_k^m + n_k^m. \]

Here

\[ H^m = \frac{1}{c_1} [0_{2 \times 1}, I_{2 \times 2}] \in \mathbb{R}^{2 \times 3}, \]
This simple update rule can be carried out independently for each transmission and only requires the addition of vectors $\mathbb{R}^6$ and multiplication of vectors in $\mathbb{R}^6$ by sparse matrices in $\mathbb{R}^{6 \times 6}$. Though this filter is currently implemented off-board, this method, because of its computational efficiency, can easily be implemented in real-time on the autonomous version of this robot [51].

4. Controller design

Similar to the complete estimator, the feedback controller is also independently derived for a transmission-sensor system. A subset of estimated actuator positions and velocities ($\hat{\mathbf{x}}_m^k$) is used in a feedback controller designed as a linear-quadratic-Gaussian (LQG) controller. LQR controllers have been used to stabilize both smooth and hybrid non-linear systems; for example, the time-varying LQR formulation (TVLQR, [52]) is often used to locally stabilize nonlinear systems about a given trajectory. Furthermore, LQR has been used to stabilize limit cycles for hybrid systems, both in full-coordinates and in transverse-coordinates using a transverse linearization [54].

In this work, since each of HAMR’s leg can exert forces greater than one body-weight [39], we can treat the relatively small contact forces as disturbances. Furthermore, since an LTI system provides an accurate representation of the transmission dynamics in air, we choose to use an infinite-horizon LQR controller. This controller minimizes the following cost function:

$$ J = \sum_{k=0}^{\infty} (\hat{\mathbf{x}}_k^m - \mathbf{x}_k^m)^T Q (\hat{\mathbf{x}}_k^m - \mathbf{x}_k^m) + (\mathbf{u}_k^m - \mathbf{u}_k^p)^T R (\mathbf{u}_k^m - \mathbf{u}_k^p), $$

where $Q \succeq 0$ and $R > 0$ are symmetric matrices that penalize deviations from the fixed point ($\mathbf{x}_k^m$, $\mathbf{u}_k^m$). We defined $Q$ and $R$ as diagonal matrices parameterized by three positive scalars ($k_p$, $k_d$, and $k_v$) that determine trade-offs between squared deviations in actuator position, velocity, and control voltage, respectively. The complete control law combines the LQR feedback rule with a feed-forward term ($\mathbf{u}_k^f = \mathbf{u}_k^m + \mathbf{u}_k^f \in \mathbb{R}^2$):

$$ \mathbf{u}_k^f = \mathbf{u}_k^m + L (\hat{\mathbf{x}}_k^m - \mathbf{x}_k^m). $$

Here $\mathbf{x}_k^m \in \mathbb{R}^6$ is the reference state, $L = (R + B^T S B)^{-1} B^T S A \in \mathbb{R}^{2 \times 4}$ is the feedback matrix, and $S$ is computed by solving the discrete-time algebraic Riccati equation [50]. The resulting linear-quadratic-Gaussian (LQG) dynamical system is formulated by combining equation (17) with the control law given in equation (19).

Intuitively, the feed-forward term is equal to the nominal voltage ($\mathbf{u}_k^f$) if the reference state is the fixed point. Furthermore, the control law in equation (19)
will stabilize the LQG system since $Q$ and $R$ are chosen to be positive-definite. In practice, the controller is used to track reference trajectories on the physical (nonlinear) legged robot, the control input ($u^e_k$) still acts to reduce the error, and ground reaction forces can be thought of as disturbances. We also augment the feed-forward term with a time varying component ($u^f_k$) that is computed via a trajectory optimization without ground contact (Note S3 (stacks.iop.org/BB/14/056001/mmedia)). This term is similar to the nominal input for a TVLQR controller about a trajectory; however, the lack of ground contact modeling makes it more of a heuristic for improving the convergence rate and reducing steady-state error.

5. Bio-inspired trajectory selection

Using the estimation and control framework described in the previous two sections (sections 3 and 4), we are now able to track arbitrary leg trajectories subject to the dynamics of the transmission. We exploit this to expand on our previous work that explored the effect of gait and stride frequency on locomotion [37]. The major challenges that limited locomotion performance in our previous studies are:

1. High leg-slip (40%–45% ineffective stance) across all stride frequencies.
2. Increased body oscillations (in roll and pitch) in the body dynamics frequency range (20 Hz–40 Hz).
3. Departure from SLIP-dynamics [55] beyond the mechanically tuned operating point close to robot z-resonance ($\sim$10 Hz).
4. Fixed (open-loop) timing between vertical and fore-aft resulting in poor or backwards locomotion (e.g. when pronking at 10 Hz).

In this work, we postulate the following four specific hypotheses to understand the underlying mechanisms behind the challenges enumerated above. These hypotheses (described below) are motivated by relevant examples from recent scientific literature, and the application of these ideas to an dexterous insect-scale system across a wide range of stride frequencies is a contribution of this work. Ultimately, we hypothesize ($H_0$) that exploring the leg trajectories described below can reveal optimized shape control parameters that enable high-performance locomotion over the entire operating range of the robot, overcoming challenges observed in our previous research [37].

5.1. Hypothesis one ($H_1$)

Template models of legged locomotion, such as SLIP, have relied on a swing-leg retraction strategy for stabilizing sagittal plane locomotion [56–60]. These results have been supported by numerous experimental studies on bipedal running [61] in humans [56, 62] and guinea fowls [8, 63], and on quadrupedal galloping in horses [64]. Expanding this approach, researchers have demonstrated an optimal retraction rate for perturbation rejection [65] and energy efficient locomotion [66]. Additionally, modeling and experimental results using large bio-inspired quadrupedal robots [65, 67] indicate that swing leg retraction can potentially mitigate the risk of slippage at heel-strike during rapid running. Therefore, we test the effect of varying leg retraction period on locomotion and hypothesize ($H_1$) that increasing the leg retraction period reduces slipping and improves locomotion performance.

5.2. Hypothesis two ($H_2$)

Upright-posture animals have been shown to modulate their normal force and vertical impulse to minimize body oscillations and maintain stable locomotion in the sagittal plane [68–71]. Similarly, studies in humans show that the above considerations are important for overcoming roll perturbations and achieving lateral stability [72, 73]. Robots employ these bio-inspired strategies [74–77] to stabilize hip height [78, 79] and control pitch oscillations [80, 81]. The underlying mechanisms either passively (mechanically) [82, 83] or actively modulate ground reaction forces [84] and impulses [85, 86]. We adapt this approach to minimize vertical, pitch, and roll body oscillations in the body dynamics frequency range, and we hypothesis ($H_2$) that increasing input lift energy, especially in the body dynamics frequency range, increases detrimental body oscillations and reduces locomotion performance.

5.3. Hypothesis three ($H_3$)

Animals of varying size and morphology [87] use energy storage and exchange mechanisms [7, 88, 89] during locomotion [10]. Numerous models explain these ubiquitous underlying mechanisms, the most popular of which is the SLIP model [55, 90–92]. Furthermore, the implications of relative stiffness [87, 93] on locomotion speed [89, 94–96], stability [97, 98] and economy [88, 99–101] are well documented across body sizes. Based on this understanding, we hypothesize ($H_3$) that increasing effective leg stiffness allows for greater energy storage and return (SLIP-like dynamics) and improves performance at higher stride frequencies.

5.4. Hypothesis four ($H_4$)

During running the body decelerates during the first half of stance, and accelerates into flight during the second half of the stance. Studies have shown that relative timing of vertical and fore-aft leg motions is important in achieving a pattern of deceleration and acceleration that results in effective locomotion [102, 103]. Given that time-of-flight will change as a function of stride frequency (due to body resonances), we hypothesize ($H_4$) that the timing between the
vertical and fore-aft leg motions that results in the best performance varies as a function of stride frequency.

5.5. Trajectory design

We distill these four hypotheses into parametric leg trajectories for the trot (figure 2(a)) and pronk (figure 2(b), supplementary video S4) gait, respectively. Each trajectory is defined by five parameters described in table 1. Here, the swing ($A_S$) and lift ($A_L$) actuator amplitudes are held constant, $T$ controls the stride frequency, and the shape parameters $S_1$, $S_2$, and $S_3$ vary as described below. For both parametric trajectories, we address $H_1$ by maintaining a constant speed during leg retraction and vary the leg retraction period as a trajectory shape control parameter $S_1$. For the trot gait, we also vary the maximum leg adduction via the shape parameter $S_2$. This modification directly varies the net energy imparted to the lift ($z$) motion addressing $H_2$. In addition $S_2$ also modulates leg stiffness (see figure S2 and Note S3) addressing $H_3$. Finally, we vary the leg adduction period as the trajectory shape control parameter $S_3$ for the pronk gait. This modification, coupled with $S_1$ from above, varies the timing between the vertical and fore-aft leg motions addressing $H_4$.

6. Experimental design, methods and metrics

This section first describes the calibration conducted before running experiments (section 6.1). We then describe the experimental procedures and apparatus for evaluating the estimator (derived in section 3) and controller (derived in section 4) performance, and for exploring the heuristic leg trajectories (developed in section 5). Finally, we define a number of locomotion performance metrics in section 6.5 that are used to quantify the effects of varying leg trajectory shape in section 8.

6.1. Calibration

A calibration was performed for each robot and single-leg before conducting all experiments. The measurement noise covariances $N_H^f$ and $N_H^u$ were computed from mean-subtracted measurements of $V^m$ and $V$, respectively, with $u^a = 0$. These means (corresponding to an initial offset) were also subtracted from subsequent measurements of $V^m$ and $V$. The velocity scaling coefficients ($\alpha$) from the mechanical current (mA) to tip velocity (mm s$^{-1}$) were computed for each actuator over the range of operating frequencies. The coefficient for each actuator was set to the value that minimized the squared-error between the mechanical current ($i_m$, equation (2)), and corresponding ground-truth leg velocity.

6.2. Estimator validation

Estimator validation was conducted on a single-leg (figure 3(a)) using the architecture shown in figure 3(c). Note that control gains ($L$) were set to zero.

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**Table 1. Heuristic trajectory design parameters.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Trot gait</th>
<th>Pronk gait</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_S$</td>
<td>Swing amplitude</td>
<td>$175$ µm</td>
<td>$150$ µm</td>
</tr>
<tr>
<td>$A_L$</td>
<td>Lift amplitude</td>
<td>$175$ µm</td>
<td>$150$ µm</td>
</tr>
<tr>
<td>$T$</td>
<td>Stride period ($\frac{\text{ms}}{\text{ms}}$)</td>
<td>$\in [150, 140, 130, 120, 110]$</td>
<td>$\in [150, 140, 130, 120, 110]$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>Shape control one</td>
<td>Leg retraction period (%$T$)</td>
<td>$\in [50, 60, 70, 80]$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Shape control two</td>
<td>Maximum leg adduction (%$A_L$)</td>
<td>$\in [-75, -50, -25, 0, 25]$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>Shape control three</td>
<td>Leg adduction period (%$T$)</td>
<td>$\in [20, 35, 50, 65, 80]$</td>
</tr>
</tbody>
</table>

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**Figure 2.** (a) Reference actuator positions for the swing (orange) and lift (blue) for the trot gait leg trajectory with $S_1 = 70$, and $S_2 = 75$. (b) The same for the pronk gait leg trajectory with $S_1 = 50$, and $S_1 = 80$. Note that $A_S$ and $A_L$ are fixed to the values given in table 1, and the smooth reference trajectories (in orange and blue) are generated by fitting a cubic-spline to the non-smooth desired trajectories (grey dashed lines).
Sinusoidal input signals \( (u_t) \) were generated at 2.5 kHz using a MATLAB xPC environment (MathWorks, R2015a), and were supplied to the single-leg through a four-wire tether. The Kalman filter (defined in section 3) estimated actuator position and velocity from the voltage measurements provided by two piezoelectric encoders at 2.5 kHz. Finally, ground truth swing and lift actuator position measurements were provided by calibrated fiber-optic displacement sensors (Philtec-D21) at the same rate.

We measured estimator performance at stride frequencies of 10, 20, 30, 40, and 50 Hz both in-air and with ground-contact. Ground contact was achieved by positioning a surface at the neutral position of the leg for the duration of the trial. Estimation error for a single actuator was quantified as \( E_{est} \), which is the N-cycle mean of the RMS error between the estimated actuator position and ground-truth measurements normalized by the peak-to-peak amplitude of the ground-truth measurements.

We also quantified estimator performance on a full-robot at frequencies of 10, 20, 30, 40, and 50 Hz using the locomotion arena shown in figure 3(b) to determine if the estimator could also be used to accurately predict leg positions \( (P, F) \). These trials were also conducted using the architecture shown in figure 3(c) with sinusoidal inputs and the control gains set to zero. Five motion capture cameras (Vicon T040) tracked the position and orientation of the robot at 500 Hz with a latency of 11 ms. A custom C++ script using the Vicon SDK enabled tracking of the leg tips in the body-fixed frame. We used a model of the transmission kinematics to map the estimated actuator position to \( P \) and \( F \), and these estimates were compared against ground truth leg position measurements provided by the motion-capture system. Performance was quantified using \( E_{est} \).

6.3. Controller validation

We also quantified controller performance on an entire robot at frequencies of 10, 20, 30, 40, and 50 Hz. Experiments were performed both in air and in the presence of ground contact using the experimental arena shown in figure 3(c) and described in section 6.2.

To determine the effectiveness of the controller performance, we quantified tracking error using \( E_{cont} \), defined as the N-cycle mean of the RMS error between the estimated and desired actuator position measurements normalized by the peak-to-peak amplitude of the desired actuator position.

6.4. Leg trajectory exploration

We also performed 400 closed-loop trials to evaluate HAMR’s performance when using the two classes of heuristic leg trajectories (section 5). These experiments used two robots whose floating-base natural frequencies are characterized in figure S1 using methods described by Goldberg et al. [37]. Two hundred trials were conducted on each robot with 100 trials for each class of heuristic leg trajectory. Each subset of one hundred trials enumerated all possible combinations of stride period \( (T) \) and shape parameters \( (S_1, S_2, \text{and } S_3) \). The 400 trials were all conducted in the locomotion arena described above. Since both robots showed similar performance, we averaged the data to compute locomotion metrics (section 6.5).

6.5. Locomotion performance metrics

We quantified the performance of the robot using the three performance metrics described below.

6.5.1. Normalized per cycle speed \((\nu)\)

This is a measure of the speed of the robot \((\nu)\) during locomotion. It is the defined as the ratio of speed achieved per step to the kinematic step length and is computed as

\[
\nu = \frac{v}{L_s n^f},
\]

where \( L_s = 4.7 \text{ mm} \) is the kinematic step length, \( n \) is the number of steps per stride for a given gait.
(\(n_{trot} = 2, n_{pronk} = 1\)) and \(f = \frac{1}{\nu}\) is the stride frequency. Intuitively, \(\nu = 1\) is the expected forward speed assuming ideal kinematic locomotion, and \(\nu > 1\) suggests that the robot is utilizing dynamics favorably to increase its stride length beyond the kinematic limits.

6.5.2. Step effectiveness (\(\sigma\))

This is a measure of the robot leg slippage during locomotion. It is defined for each leg as one minus the ratio of leg-slip to the kinematic step length. We consider leg-slip to be the total distance a single leg travels in the direction opposite to the robot heading in the world frame. We present an average value for all four legs computed as

\[
\sigma = 1 - \frac{1}{4L_i} \sum_{i=1}^{4} \int_{\zeta} |\nu'_x(t)| dt, \tag{21}
\]

where \(\nu'_x\) is the x-velocity of the \(i\)th leg in the world-fixed frame, and \(\zeta\) is the set of times within a step for which \(\nu'_x\) is in the opposite direction as the robot heading. Intuitively, \(\sigma = 1\) indicates no slipping while \(\sigma = 0\) indicates continuous slipping (i.e. no locomotion of the robot).

6.5.3. Locomotion economy (\(\epsilon\))

This is a measure of the the robot’s COT [104]. This is defined as the ratio of the robot’s mechanical output power to the total electrical power consume and is quantified as:

\[
\epsilon = \frac{mg\nu_x}{\sum_{i=1}^{3} \frac{1}{4} \int_0^T \mu(t)V^m(t) dt}, \tag{22}
\]

where \(m = 1.43\) g is the mass of the robot and \(g = 9.81\) m s\(^{-2}\) is the acceleration due to gravity. Intuitively, lower values of \(\epsilon\) indicate poor conversion of the input electrical power into mechanical output, suggesting ineffective locomotion performance.

6.6. Open-loop control trajectory comparison

Finally, we also conducted the following open-loop experiments to serve as a baseline for the experiments described in section 6.4.

6.6.1. Coupled sinusoids

The RMS amplitude for each sinusoidal drive voltage was equal to the average of the RMS voltages delivered to all eight actuators during the fastest trial at a particular stride period. This control experiment did not discriminate between voltages delivered to the lift and swing DOFs and is therefore referred to as the coupled configuration.

6.6.2. Decoupled sinusoids

The RMS amplitude for the four lift (and four swing) actuators was equal to the average RMS voltage delivered to the lift (and swing) actuators during the fastest trial at a particular stride period, respectively.

The voltages delivered to the lift and swing actuators were individually computed, and therefore, this is referred to as the decoupled configuration.

7. Estimator and controller performance

This section summarizes our results related to the quantification of estimator and controller performances. In particular, we evaluate the accuracy of both the linear approximation (described in section 3.1) of the transmission model and the treatment of ground contact as a perturbation (described in section 4).

7.1. Estimator

The performance of the estimator is shown in figure 4 with estimation errors for a representative trial in air and on the ground shown in supplementary figure S4. For the trials in air (figure 4(a)), the mean normalized estimation error in actuator position (blue, one transmission, one robot) and leg position (orange, four transmissions, one robot) as a function of stride frequency. (b) Mean and standard deviation for normalized estimator error with ground contact in actuator position (blue, one transmission, one robot) and leg position (orange, eight transmissions, two robots) for the swing DOF (top) and the lift DOF (bottom). All values of normalized estimation error for (a) and (b) are computed across 15 cycles.

![Figure 4](image-url)
Similarly, actuator position error (blue) is low when subject to approximated ground-contact. The normalized swing (figure 4(b)) and lift (figure 4(c)) actuator position errors are between 5%–10% and 8%–16%, respectively. We suspect that the lift position errors are higher because the process model does not capture the effect of (1) perturbations from ground contact and (2) serial compliance between the actuator and mechanical ground [105]. Nevertheless, these errors are still relatively small, indicating that the Kalman filter effectively averages the sensor measurement that registers contact with a linear prediction that does not drift.

Finally, we find that the normalized leg position error (orange) with ground contact is higher than normalized actuator position-error (blue). The leg-x error (\(l_x\), figure 4(b)) ranges from 11% to 24%, and leg-z error ranges (\(l_z\), figure 4(c)) from 23% to 29%. The most likely cause of this is the serial compliance in the transmission—a common problem in flexure-based devices [39, 105]. This serial compliance alters the kinematics of the transmission by effectively adding un-modeled DOFs between the actuators and leg and changes the assumed one-to-one mapping between actuator and leg positions.

### 7.2. Controller

The performance of the controller is shown in figure 5 with tracking errors for a representative trial in air and on the ground shown in supplementary figure S5. For the trials in air (figure 4(a)), the mean normalized estimation error in actuator position increases from 5% at 10 Hz to 15% at 50 Hz for the swing DOF and from 5% at 10 Hz to 11% at 50 Hz for the lift DOF. This demonstrates the linear approximation of the transmission dynamics is sufficient for control in the absence of ground contact. Moreover, the normalized tracking error (figure 5(b)) for both the swing and lift DOFs when running is also small, and it increases from 6% at 10 Hz to 16% at 50 Hz. This indicates that treating ground contact as a perturbation does not significantly reduce tracking performance. Finally, a likely reason for the increase in tracking error as a function of stride frequency is that the high-frequency components in the heuristically designed leg trajectories become harder to track as they approach the robot’s transmission resonant frequencies (between 80 Hz–100 Hz, [106]).

### 8. Locomotion performance

The average value for each locomotion performance metric described in section 6.5 are plotted as a function of the shape control parameters (table 1) at all five tested stride frequencies (10 Hz–50 Hz) in figure 6. We first summarize the robot’s locomotion performance for the trot gait, validate hypotheses \(H_1\) and \(H_5\), and invalidate hypothesis \(H_4\). We then summarize performance for the pronk gait, refute hypothesis \(H_1\), and validate hypothesis \(H_4\).

#### 8.1. Trot gait performance summary

As shown in figure 6(a), we are able to achieve locomotion over a wide range of speeds (43 mm s\(^{-1}\)–278 mm s\(^{-1}\) or 0.95 BL s\(^{-1}\)–6.17 BL s\(^{-1}\), \(n = 200\) trials, \(N = 2\) robots) by varying stride frequency and the shape control parameters. We also measure step effectiveness for the above gaits ranging from 0.25 to 0.91 (figure 6(a)). In addition, we find that locomotion economy (figure 6(a)) varies nearly four-fold (0.08–0.30) and shows a strong dependence on shape control parameters both within and across frequencies. The resulting cost of transport (COT) values range from 3.33–13.14, and are some of the lowest measured on this platform [35, 37]. Finally, we note that COT increases with frequency while maintaining a trot, supporting the hypothesis that the preferred gait varies as a function of running speed [107]. The best and worst performing trials are visualized in supplementary video S2.

#### 8.2. \(H_4\)–trot gait

For all stride frequencies, a higher leg retraction period results in increased step effectiveness (C, figure 6(a)). Leg retraction period, however, is only positively correlated with per-cycle velocity at high stride frequencies (A, figure 6(a)). Finally, a higher leg retraction period results in lower locomotion economy at all stride frequencies (E, figure 6(a)). These trends support our initial hypothesis \((H_4)\) that increasing leg retraction period increases step effectiveness by decreasing slipping. However, step effectiveness is only a good predictor of speed at high stride frequencies (D2, figure 6(a)), and the two are uncorrelated at low stride frequencies (D1, figure 6(a)). This is because the body dynamics (figure S1) have a dominating effect on speed at lower stride frequencies. These dynamics, however, are attenuated at higher stride frequencies, and, therefore, speed in those regimes is largely determined by the magnitude of foot slipping [37]. This negative correlation between locomotion economy and leg retraction period also indicates that the energetic cost of tracking the high-speed leg protraction might offset the benefit of mitigating leg slip. Finally, our results corroborate previous findings [65–67] that imply the existence of preferred values of leg retraction period that minimize foot slippage and economy, respectively. Moreover, we find that these values are a function of the stride-frequency dependent dynamics of the robot.

#### 8.3. \(H_2\) and \(H_3\)–trot gait

For all stride frequencies, higher maximum leg adduction results in both higher step effectiveness (C, figure 6(a)) and higher per-cycle velocity (B, figure 6(a)). These trends refute our initial hypothesis \((H_2)\) that increasing the maximum leg adduction
motion and supporting our initial hypothesis (H₁) that increasing leg adduction increases the relative leg stiffness for interm leg adduction (independent of leg retraction) result in ineffective locomotion. This note that actuator per-cycle energy consumption is independent of the stride frequency and the gait shape control parameters, and, as a consequence, the contour maps of ε mirror that of ν. The best and worst performing trials are visualized in supplementary video S3.

8.5. H₁—pronk gait
We find that the lowest leg retraction period results in the highest per-cycle velocity (F, figure 6(b)) and locomotion economy (H, figure 6(b)) across all stride frequencies. This matches our intuition that rapid leg swing retraction during stance is key to maximizing the net forward impulse imparted to the robot. Furthermore, we do not see a clear trend in the dependence of step effectiveness on leg retraction period (G, figure 6(b)); however, we again see that step effectiveness is a good predictor of normalized per-cycle speed at higher stride frequencies. These trends refute our initial hypothesis H₁ that increasing leg retraction period reduces leg slip and therefore results in improved performance.

8.6. H₂—pronk gait
A high leg adduction period and low swing retraction period results in fast forward locomotion (F, figure 6(b)), high step effectiveness (G, figure 6(b)), and high locomotion economy (H, figure 6(b)) for stride frequencies from 20 Hz–50 Hz. Similarly, a low leg adduction period and high leg retraction period results in fast backwards (enclosed by a purple polygon) locomotion and high locomotion economy. Finally, intermediate values of leg adduction (independent of leg retraction) result in ineffective locomotion. This supports our initial hypothesis (H₂) that the timing between vertical and fore-aft leg motions is crucial in determining locomotion performance and direction, and matches similar observations from previous studies [102, 103]. In contrast, we observe a reversal in the trends described above (I, figure 6(b)) at a stride frequency of 10 Hz where the robots mechanical z-resonance results in long flight phases that favor a shorter leg adduction period.

9. Effective locomotion performance across dynamic regimes
We analyze the best performing trials (figure 7) to test our final hypothesis (H₃) that closed-loop trajectory modulation enables high-performance locomotion across stride frequencies. Using speed as the primary metric to facilitate a comparison with previous results from [37], we define the best performing trial as the one with the highest normalized per cycle speed (ν).
at each frequency for the trot and pronk, respectively. However, we also plot step effectiveness ($\epsilon$) and locomotion economy ($\sigma$) for the best performing trials to consider multi-dimensional robot performance.

For the trot gait, we find that closed-loop heuristic trajectories allow the robot to maintain high speed locomotion across all stride frequencies (figure 7(a)). This is in contrast with the open-loop results from [37] and the coupled sinusoidal trajectories (section 6.6.1) where the robot suffers from poor performance in intermediate frequency regimes (15 Hz–35 Hz, supplementary video S1). However, we find that there is minimal difference in robot speed when using either the closed-loop heuristic leg trajectories or the decoupled sinusoidal trajectories (section 6.6.2). A similar trend is observed with locomotion economy (figure 7(c)); however, the closed-loop heuristic trajectories enable higher step effectiveness at all stride frequencies greater than 10 Hz (figure 7(b)). These results suggest that, while the shape of leg trajectories is important for effective locomotion using the trot gait in the body dynamics regime (15 Hz–35 Hz), the distribution of energy between the leg vertical and fore-aft motion achieved via leg shape modulation is the significant consideration at operating conditions where the dynamics are neither mechanically tuned (10 Hz) nor attenuated (40 Hz–50 Hz).

Similarly, we also find that closed-loop heuristic trajectories allow the robot to maintain speed across all stride frequencies (figure 7(d)) when using a pronk gait. This is in contrast with the open-loop results from [37], the coupled sinusoidal trajectories, and
the decoupled sinusoidal trajectories where the robot suffers from poor performance between 5 Hz–25 Hz (supplementary video S1). On the other hand, closed-loop heuristic trajectories enable higher step effectiveness (figure 7(e)) and locomotion economy (figure 7(f)) across all stride frequencies compared to coupled input matched open-loop trajectories. This validates hypothesis $H_0$, indicating that leg trajectory modulation enables high performance locomotion across stride frequencies.

10. Conclusion and future work

We have presented a computationally efficient framework for proprioceptive sensing and control of leg trajectories on a quadrupedal microrobot. We used this capability to explore two parametric leg trajectories designed to test a series of hypotheses investigating the influence of leg slipping, stiffness, timing, and energy on locomotion performance. This parameter sweep resulted in an experimental performance map that allowed us to select control parameters and determine a leg trajectory that maximized performance at a desired gait and stride frequency. Using these parameters, we recovered effective performance over a wide range of stride frequencies, achieving locomotion that is robust to perturbations from the robot’s body dynamics [108].

Specifically, for the trot gait, we demonstrated that maximizing robot speed depends on minimizing slipping at high stride frequencies and leveraging favorable dynamics at low and intermediate stride frequencies. We found that the mechanism for doing either was modulating leg trajectory shape, and consequently, input energy. In addition, we were able to increase energy storage and return by modulating leg stiffness, which resulted in faster locomotion. Furthermore, we found that leg timing determined performance for the pronk gait and allowed for rapid locomotion in the forward or backwards directions.

As potential next steps towards improving the robot’s state estimation, we plan to explicitly address the hybrid nature of the robot’s underlying dynamics. Such an effort would require an appropriate contact sensor and a modification of the current estimation and control framework, and in principle could result in improved tracking performance. Moreover, we aim to use this low-level controller in conjunction with the trajectory optimization scheme described by Doshi et al [109] to design feasible leg trajectories that optimize a given cost (e.g. speed, COT, etc) at a particular operating condition. This can automate the challenging task of designing appropriate leg trajectories for a complex legged system and result in better locomotion performance. Finally, we can use this controller to ensure accurate tracking of the leg trajectories during a variety of locomotion modalities including swimming [110] or climbing [111] with HAMR.

In addition to the planning and control efforts discussed above, the small footprint and mass of the sensors combined with the computational efficiency of the estimation and control scheme makes our approach suitable for future implementation on the autonomous version of HAMR [51]. We can also use the results from this work to inform future mechanical design decisions. For example, increasing the
transmission resonant frequencies [106] can increase control authority and enable improved leg trajectory control at stride frequencies higher than those tested in this work (>50 Hz). Ultimately, our results suggest that HAMR could be a strong candidate platform for systematically testing hypotheses about biological locomotion such as the effect of varying leg trajectories on locomotion [112].

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